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OFFICE OF THE DEPUTY PRINCIPAL

ACADEMICS, STUDENT AFFAIRS AND RESEARCH

UNIVERSITY EXAMINATIONS

2019 /2020 ACADEMIC YEAR

FIRST YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF BACHELOR OF SCIENCE CS/ASC

COURSE CODE: MAT 314

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: 5th DEC 2019 **TIME: 9AM-12PM**

INSTRUCTION TO CANDIDATES

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MAT 314: ORDINARY DIFFERENTIAL EQUATIONS I

STREAM: BSc (CS&ASC) **DURATION: 3 Hours**

INSTRUCTION TO CANDIDATES

i. *Answer **ALL** questions from **section A** and any **THREE** from **section B***

ii. *Do not write on the question paper.*

SECTION A (31 MARKS): Answer all questions in this section.

a) Define the following:
   i) Exact differential equation
   ii) Singular solution. (4 marks)

b) Show that the differential equation $(ax+by+c)dx-((A+1)x+By+C)dy=0$ is an exact differential equation iff $A+b=1$ (3 marks)

c) Solve the equation $xy'=xe^{-\frac{y}{x}}+y$. (3 marks)

d) Solve the linear fractional equation $y'=\dfrac{x+y-3}{x-y-1}$ (4 marks)

e) Is $e^x\cos 2x$ a solution to $y''+4y=0$? (2 marks)

QUESTION TWO (15 marks)

a) Find the particular solution for the differential equation $\dfrac{d^2y}{dx^2}-y=0$, $y(0)=1$ and $y'(0)=1$. (4 marks)

b) Use appropriate method to obtain the solution of $x\dfrac{d^3y}{dx^3}-2\dfrac{d^2y}{dx^2}=0$ (5 marks)

c) Find the orthogonal trajectory of $x^2-y^2=1$ (4 marks)

d) Solve $y''+5y'+6y=0$. (2 marks)

SECTION B: 39 MARKS (ATTEMPT ANY THREE QUESTIONS)

QUESTION THREE (13 MARKS)

a) Obtain the solution of the differential equation $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}=3x$ (5 marks)

b) A thermometer reading 100^0F is placed in a pan of oil maintained at 10^0F. What is the temperature of the thermometer when $t=10\text{sec}$, if its temperature is 60^0F when $t=4\text{sec}$? (8 marks)

QUESTION FOUR (13 MARKS)

Solve ;

a) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} - 4y = 2\cos 3x$. (5 marks)

b) $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^3$. (5 marks)

c) $\frac{dy}{dx} + y\cos x = \cos x$ (3 marks)

QUESTION FIVE (13 MARKS)

a) Consider the initial value problem $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = 0$, $y(0) = 1$, and $y'(0) = -1 + 2\sqrt{3}$

i) Find its solution (4 marks)

ii) Write the solution in the form $Ce^{\alpha x}\cos(\beta x - \alpha)$ (3 marks)

b) Solve the equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{1}{1+e^{2x}}$. (5 marks)

QUESTION SIX (13 MARKS)

a) Ignoring the air resistance, a sailboat starting from rest accelerates $\left(\frac{dv}{dt}\right)$ at a rate proportional to the differences between the velocities of the wind and the boat. Write the velocity as a function of time if the wind is blowing at $20\,{}^{m}/_{s}$ and after one second the boat is moving at $5\,{}^{m}/_{s}$. Assume that the boat started from rest.

(8 marks)

b) If the integrating factor of the differential equation $M(x,y)dx + N(x,y)dy = 0$ is a function of x only, obtain the expression for integration factor. (5 marks)

QUESTION SEVEN (13 MARKS)

a) In the following problem, find the integrating factor of $3x^2ydx - 2x^3dy = 0$ (5 marks)

b) A cup of coffee (temperature = 190°F) is placed in a room whose temperature is 70°F. After five minutes, the temperature of the coffee has dropped to 160°F. How many more minutes must elapse before the temperature of the coffee is 130°F? (8 marks)