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OFFICE OF THE DEPUTY PRINCIPAL
ACADEMICS, STUDENT AFFAIRS AND RESEARCH

# UNIVERSITY EXAMINATIONS

# 2021 /2022 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER REGULAR EXAMINATION

## FOR THE DEGREE OF BACHELOR OF EDUCATION ARTS AND SCIENCE

COURSE CODE: MAT 204

COURSE TITLE: LINEAR ALGEBRA II

DATE: $2^{ND}$ FEB 2022 TIME: 2.00PM -5.00PM

## INSTRUCTION TO CANDIDATES

- SEE INSIDE

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## INSTRUCTIONS TO CANDIDATES

i. *Answer ALL Questions from **section A** and any **THREE** from **section B**.*

ii. *Do not write on the question paper.*

## SECTION A (31 Marks)

**Answer ALL questions in this section.**

### Question One (16 Marks)

a) Verify that the following is an inner product in $P_2$

$$\langle p(x), q(x)\rangle = a_0b_0 + a_1b_1 + a_2b_2$$

where $p(x) = a_0 + a_1x + a_2x^2$ and $q(x) = b_0 + b_1x + b_2x^2$. (4 marks)

b) Transform the set $S = \{1, x, x^2\}$ which is a basis for $P_2$ into an orthonormal basis using the Gram-Schimdth orthonormalization process with respect to the integral inner product on $P_2$ defined as

$$\langle p(x), q(x)\rangle = \int_{-1}^{1} p(x)q(x)dx$$ (5 marks)

c) Given $T:\mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear transformation defined as $T(x, y) = (2x - 3y, x + y)$ and $B = \{(1, 2), (2, 3)\}$, $B' = \{(1, 3), (1, 4)\}$ are both bases for $\mathbb{R}^2$. Find

i) A i.e matrix of representation of T with respect to basis B.

ii) Transition matrix P from $B'$ to $B$. Hence use P and A to obtain $A'$ (i.e. matrix of representation of T with respect to $B'$). (7 marks)

### Question Two (15 Marks)

a) Let $A = \begin{pmatrix} 2 & -3 \\ 4 & -5 \end{pmatrix}$ use the eigen value method to derive an explicit formula for $A^n$ and solve the system of differential equations $\frac{dx}{dt} = 2x - 3y$ and $\frac{dy}{dt} = 4x - 5y$ given that x = 7 and y = 13 when t = 0. (6 marks)

b) Identify the curve which is represented by the following quadratic equation by first putting it into standard conic form, $x^2 + 2xy + y^2 - x + y = 0$ (6 marks)

c) Find the inverse of the matrix $A=\begin{pmatrix}1 & 4 & -3\\ 0 & 3 & 1\\ 0 & 2 & -1\end{pmatrix}$ by making use of the Cayley-Hamilton theorem.

(3 marks)

## SECTION B (39 Marks)

**Answer any THREE questions.**

### Question Three (13 Marks)

a) Consider the vector space of polynomials with inner product defined by $\langle f(x), g(x)\rangle = \int_{-1}^{1} f(x)g(x)dx$ and the polynomials $f(x)=x$ and $g(x)=1+x$. Find

i) $\langle f(x), g(x)\rangle$

ii) $\|f\|$

iii) $\|g\|$

iv) Normalize f and g. (6 marks)

b) State and prove the Cauchy - Schwarz inequality. (7 marks)

### Question Four (13 Marks)

a) For the matrix $A=\begin{pmatrix}2 & 0 & -2\\ -1 & 2 & -1\\ -2 & 0 & 2\end{pmatrix}$

i) write down the characteristic polynomial

ii) write down the characteristic equation

iii) find the eigen values and eigen vectors corresponding to each eigen value.

iv) find the basis for each eigen space. (10 marks)

b) Find the minimal polynomial $m(\lambda)$ of the matrix

$$A=\begin{pmatrix}\alpha & \beta\\ 0 & \alpha\end{pmatrix}$$

(3 marks)

### Question Five (13 Marks)

a) If $\lambda_1$ and $\lambda_2$ are eigen values of a symmetric matrix A associated with eigen vectors $v_1$ and $v_2$ respectively, show that $v_1$ and $v_2$ are orthogonal. (4 marks)

b) If $v_1, v_2, \cdots, v_n$ are eigenvectors associated with distinct eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of a matrix $A_{n\times n}$ then show that the set $\{v_1, v_2, \cdots, v_n\}$ is linearly independent. (6 marks)

c) Determine if $B = \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & \frac{-2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}$ is orthogonal matrix (3 marks)

**Question Six (13 Marks)**

a) Let V be the vector space of polynomials over $\mathbb{R}$ and define $\langle f(t), g(t)\rangle = \int_0^1 f(t)g(t)dt$.

Find the angle $\theta$ between u and v if $u = 2t - 1$ and $v = t^2$. (6 marks)

b) Show that a square matrix $A_{n\times n}$ is diagonalizable if and only if A has n linearly independent eigenvectors. (7 marks)

**Question Seven (13 Marks)**

a) Consider the bases of $\mathbb{R}^3$; $B = \{(1,0,0), (0,1,0), (0,0,1)\}$ and $B' = \{(1,1,1), (1,1,0), (1,0,0)\}$.

i) Find the transition matrix $P$ from $B'$ to $B$.
ii) Find the transition matrix $Q$ from $B$ to $B'$
iii) Verify that $Q = P^{-1}$
iv) Compute coordinate matrix of $w$ with respect to $B'$ where $w = (-5,8,5)$.

(9 marks)

b) Let $T: V \to V$ be a linear transformation. Let $A$ be matrix of $T$ with respect to $B$ and $A'$ be matrix of $T$ with respect to $B'$. *Show that* $A' = P^{-1}AP$ *where* $P$ *is* transition matrix from $B'$ *to* $B$. (4 marks)

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